Fractals

Outcomes



The Mandelbrot Set

 The Mandelbrot Set is the set of numbers resulting from repeated iterations of the complex function:

$$Z_n = Z_{n-1}^2 + C$$
 with the initial condition $Z_0 = 0$

• $C = x_0 + iy_0$ belongs to the Mandelbrot set if |Z| converges.

$$Z = x + iy \quad \Rightarrow \quad Z^2 = x^2 + i2xy - y^2$$





The Mandelbrot Set cont.

Separating out the real and imaginary parts gives:

$$Z^{r} = x^{2} - y^{2} + x_{0}$$

$$Z^{i} = 2 xy + y_{0}$$

• Take the threshold value as:

$$\left|Z\right|^2 \ge 4.0$$

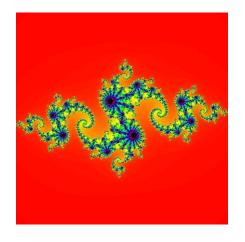
- Set the maximum number of iterations $\,N\,$
 - Assume that $_Z$ does not diverge at higher values of $\,N$





The Julia Set

- Similar algorithm to Mandelbrot Set
- Starting coordinates x_0 and y_0 represent fixed point from inside the Mandelbrot set









Visualisation

To visualise a Mandelbrot/Julia set:

- Represent the complex plane as a 2D grid.
- Calculate number of iterations N for complex numbers C corresponding to points on the grid.
- \bullet Convert the value of N to a colour and plot this on the grid.





Parallelisation

- Values for each coordinate depend only on the previous values at that coordinate.
 - decompose 2D grid into equally sized blocks
 - no communications between blocks needed.
- Don't know in advance how much work is needed.
 - number of iterations across the blocks varies.
 - work dynamically assigned to workers as they become available.

Implementation

- Split the grid into blocks:
 - each block corresponds to a task.
 - master process hands out tasks to worker processes.
 - workers return completed task to master.





Example: Parallelisation on 4 CPUs

CPU 2

master

CPU 1

8

5

6

3

7

1

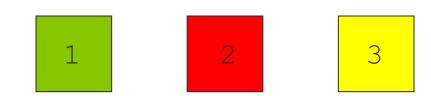
Х

У

workers

CPU 3

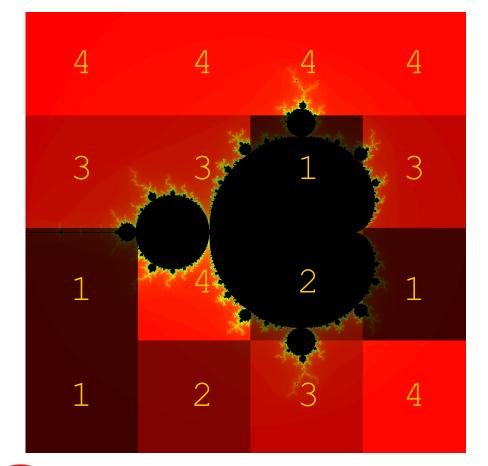
CPU 4







Parallelisation cont.



- taskfarm run on 5 CPUs

 master
 workers
- total number of tasks = 16







Example results –

fixed number of workers

Example results for the default image size (768×768 pixels), fixed number of iterations (5000), fixed number of workers (16) and varying number of tasks :

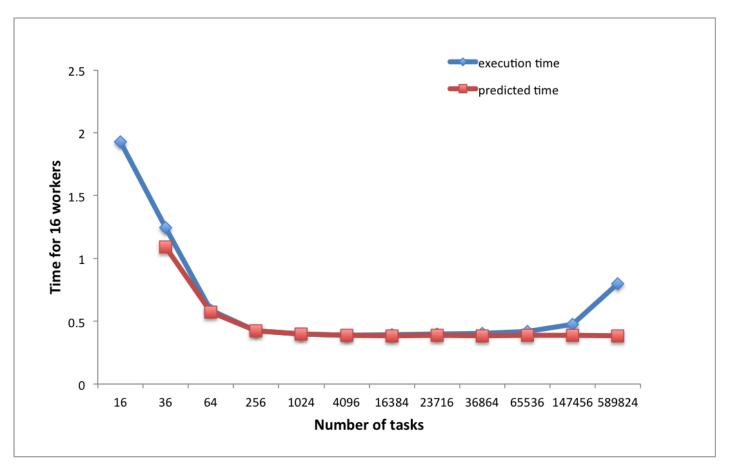
Number of Tasks (Task Size)	Time (s)	Load Imbalance Factor
16 (192 × 192)	1.93	5.034
64 (96 × 96)	0.59	1.501
256 (48 × 48)	0.43	1.108
4096 (12 × 12)	0.4	1.017
36864 (4 × 4)	0.4	1.003
147456 (2 × 2)	0.47	1.017
589824 (1 × 1)	0.80	1.006

Table 2: Example execution Times for 16 workers and varying number of Tasks.





Results cont.







Example results –

fixed number of tasks

Example results for the default image size (768×768 pixels), fixed number of iterations (5000), 64 tasks of the size of 96 x 96 pixels and varying number of workers :

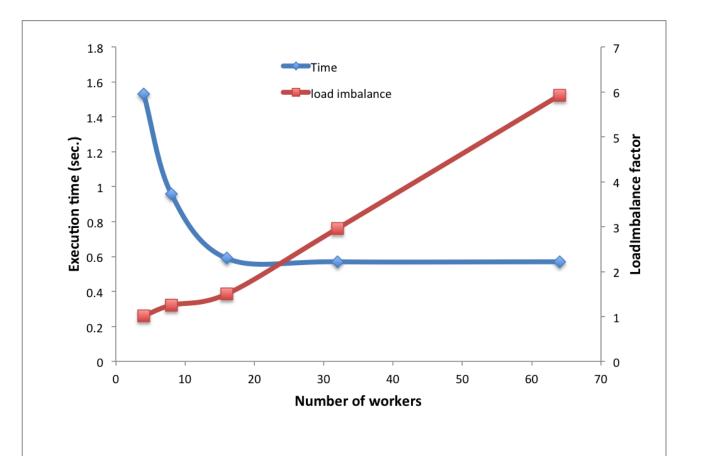
Workers	Time (s)	Avg. Workload	Max Workload	Min Workload	Load Imbalance Factor
4	1.52	124505763	126124383	122696852	1.01
8	0.96	62252881	77744803	51117022	1.25
16	0.59	31126440	46737752	10968369	1.50
32	0.57	15563220	46114456	67614	2.96
64	0.57	7781610	46089216	9246	5.92

Table 1: Example Run-Times for different number of workers and their Avg/Min/Max Workloads.





Results cont.









TASK FARMS

- Also known as the master/worker pattern
- Allows a master process to distribute work to a set of workers processes.
- Can be used for other types of tasks but it complicates the situation and other patterns may be more suitable for implementing.
- Master process is responsible for creating, distributing and gathering the individual jobs.





TASKS

- Units of work
- Vary in size, do not have to be of consistent execution time. If execution times are known it can help with load balancing.

QUEUES

- Master generates a pool of tasks and puts them in a queue
- Workers assigned task from queue when idle





LOAD BALANCING

- How a system determines how work or tasks are distributed across workers (processes or threads)
- Successful load balancing avoids idle processes and overloading single cores
- Poor load balancing leads to under-utilised cores, reducing performance.





COST

- Increasingly important
- Finite budgets require optimal use of resources requested.
- Load balancing is just one method of ensuring optimal usage and avoiding wasting resources.
- More power and resources do not necessarily mean improved performance.
- Always ask is it necessary to run this on 4000 cores or could it be run on 2000 more efficiently?



