Fractals exercise

Investigating task farms and load imbalance

Reusing this material

Aims

- Explore how the granularity of tasks impacts performance
  - Trade-off between the amount of parallelism (number of parallel tasks) and amount of communication (size of tasks)

- Consider issues surrounding load balance
  - Remember the runtime of the code is determined by the slowest running task – so we want work to be as evenly distributed as possible
  - The exercise introduces a Load Imbalance Factor (LIF) which illustrates how much faster your code could run if the load was evenly distributed

What are fractals?

Ideas behind the Mandelbrot and Julia sets

The Mandelbrot Set

- The Mandelbrot Set is the set of numbers resulting from repeated iterations of the complex ($i = \sqrt{-1}$) function:

$$Z_n = Z_{n-1}^2 + C$$ with the initial condition $$Z_0 = 0$$

- $C = x_0 + iy_0$ belongs to the Mandelbrot set if $|Z_n|$ remains bounded i.e. does not diverge

$$Z_n = x_n + iy_n \, , \, Z_n^2 = (x_n^2 - y_n^2 + 2ix_n y_n) \, , \, |Z_n|^2 = (x_n^2 + y_n^2)$$

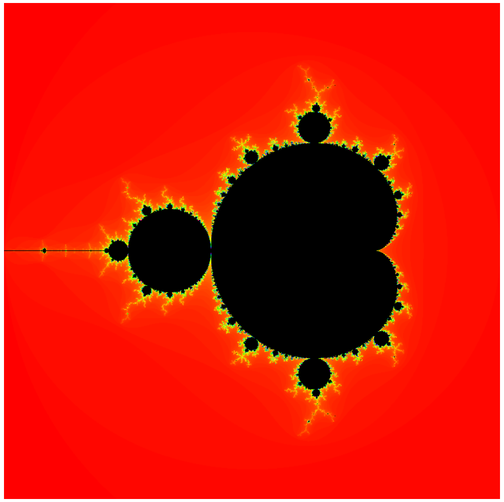
The Mandelbrot Set cont.

- Separating out the real and imaginary parts gives:

$$Z_n = Z_n^r + iZ_n^i$$

$$Z_n^r = x_{n-1}^2 - y_{n-1}^2 + x_0$$

$$Z_n^i = 2x_{n-1}y_{n-1} + y_0$$



- Take the threshold value as:

$$|Z|^2 \geq 4.0$$

- Set the maximum number of iterations to N_{max}

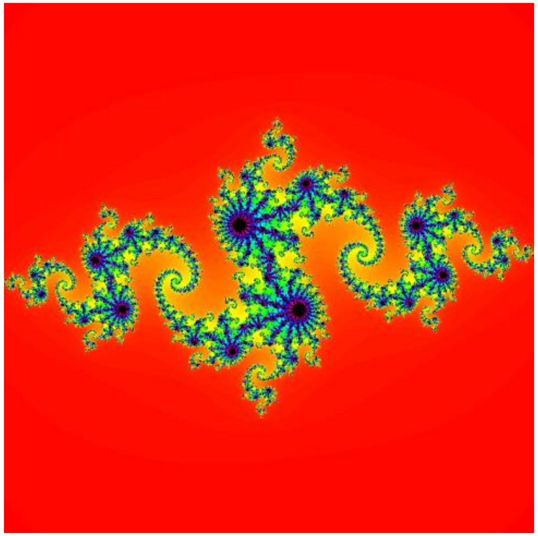
  - Assume that Z does not diverge at higher values of N_{max}

The Julia Set

- Similar algorithm to Mandelbrot Set – recall:

$$Z_n = Z_{n-1}^2 + C \,, \quad C = x_0 + iy_0 \,, \quad Z_0 = 0$$

- There are an infinite number of Julia sets, parameterised by a complex number *C*



$$Z_n = Z_{n-1}^2 + C \,, \quad Z_0 = x_0 + iy_0$$

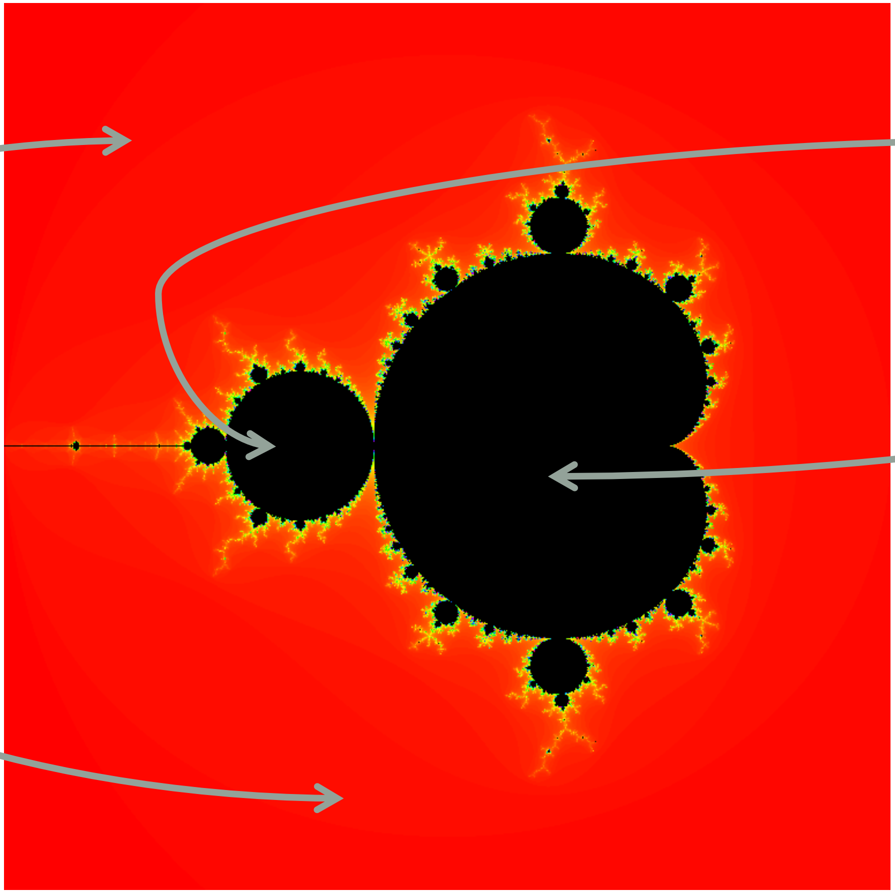
  - for example, $C = 0.8 + i\,0.156$

Visualisation

To visualise a Mandelbrot/Julia set:

- Represent the complex plane as a 2D grid where complex numbers correspond to points on the grid (x, y)
- Calculate number of iterations N for the series to diverge (exceed the threshold) for each point on the grid
  - If it does not diverge, $N = N_{max}$
- Convert the value of N to a colour and plot this on the grid

Mandelbrot Set



Very quick to compute

Very slow to compute

Parallel implementation

How do we parallelise computation of these fractals?

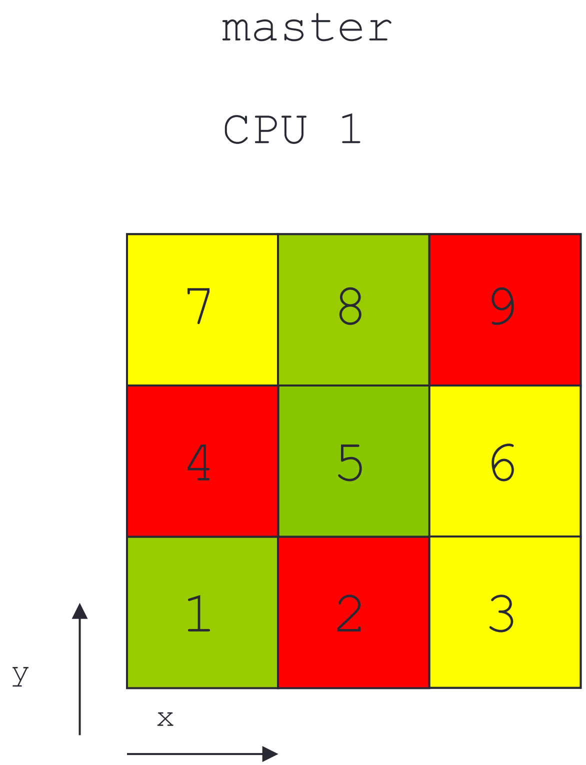
Parallelisation

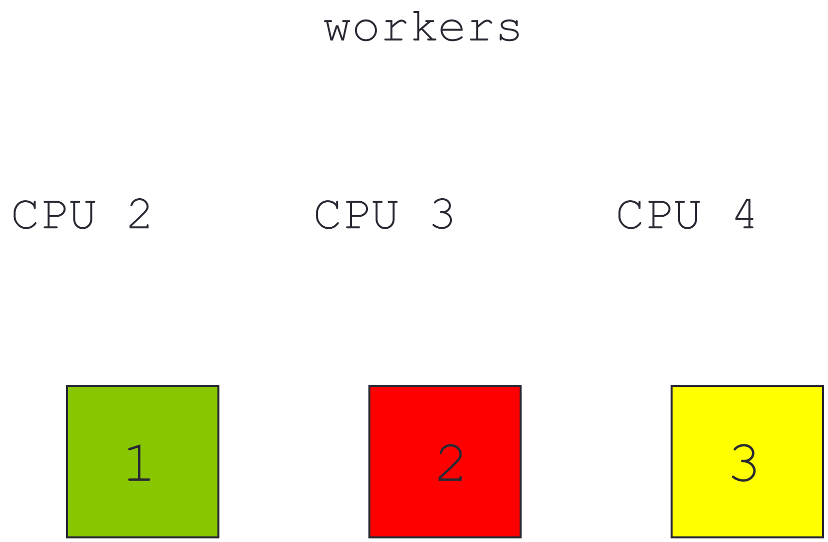
- Values for each coordinate depend only on the previous values at that coordinate.
  - decompose 2D grid into equally sized blocks
  - no communications between blocks needed.
- Don’t know in advance how much work is needed.
  - number of iterations across the blocks varies.
  - work dynamically assigned to workers as they become available.

Implementation

- Split the grid into blocks:
  - each block corresponds to a task.
  - **master** process hands out tasks to **worker** processes.
  - workers return completed task to master.

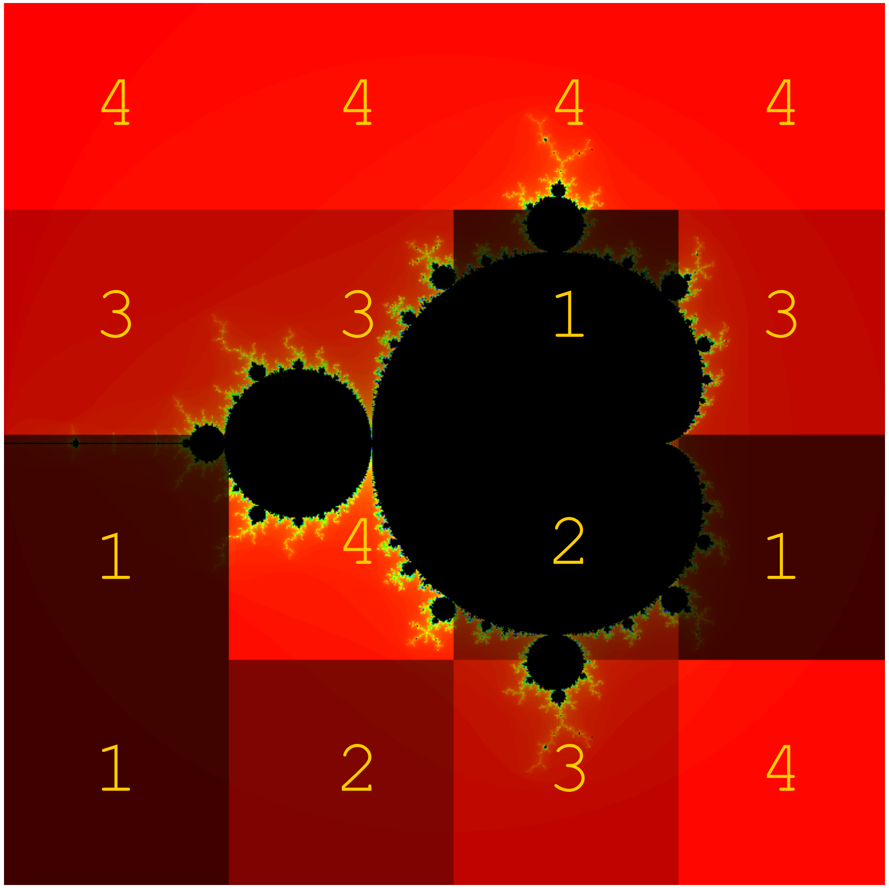
Example: Parallelisation on 4 CPUs





- In diagram, colour represents which worker did the task
  - number gives the task id
  - tasks scan from left to right, moving upwards

Parallelisation cont.



- in supplied code
  - shading represents worker
  - here we have added worker id as a number by hand
- e.g. taskfarm run on 5 CPUs
  1 master
  4 workers
- total number of tasks = 16

Notes about the exercise

Exercise

- You are supplied with source code etc.

- Compile and run on the machine
  - Visualise results

- Quantify performance results

- For a fixed number of workers
  - improve load balance by increasing number of tasks (decrease size)
  - compute LIF to estimate minimum achievable runtime
  - is this minimum ever reached?

Exercise outcomes

What do the timings tell us about HPC machines?

Example results (fixed number of workers)

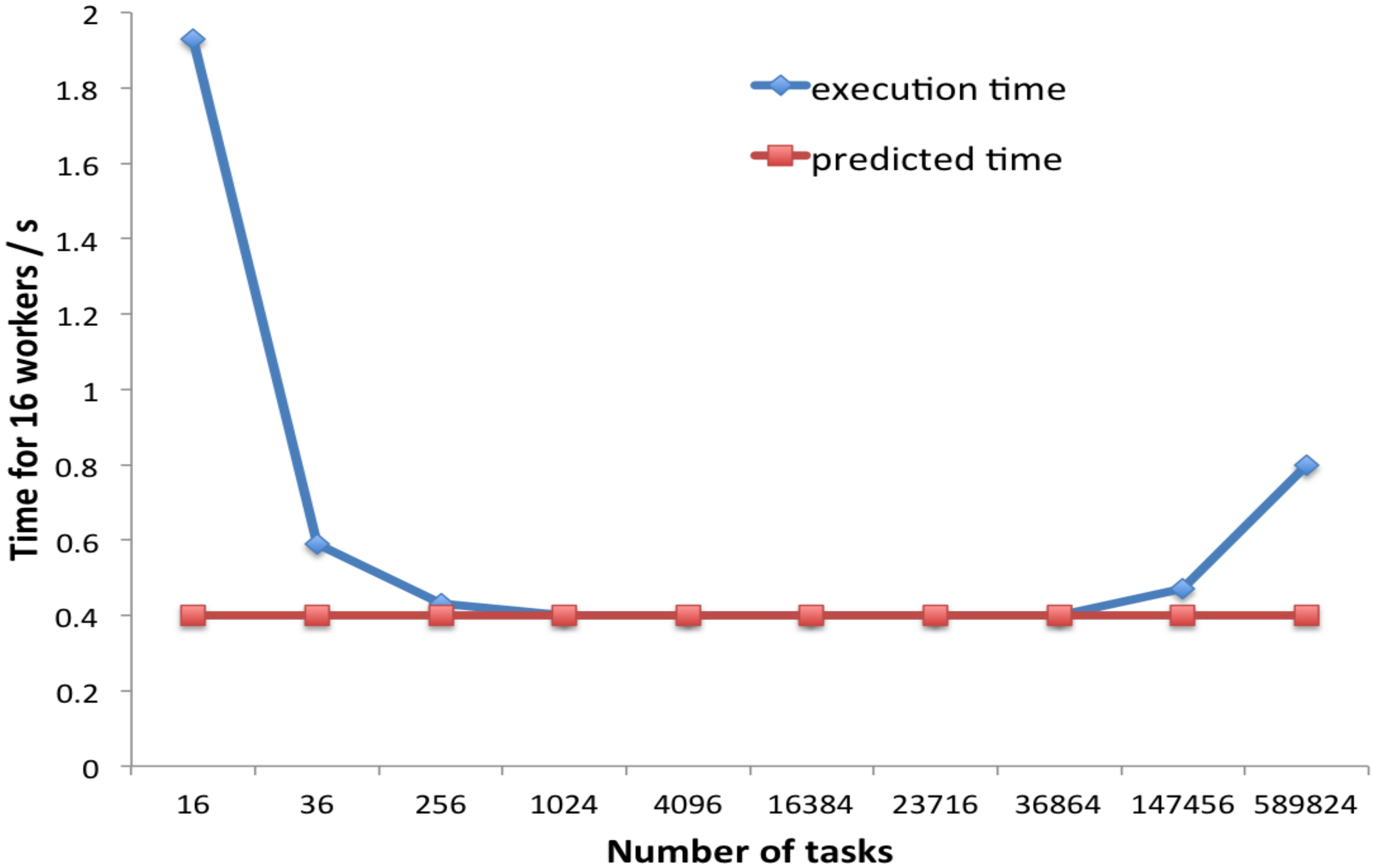
Example results for the default image size (768 × 768 pixels), fixed number of iterations (5000), fixed number of workers (16) and varying number of tasks :

Number of Tasks (Task Size)	Time (s)	Load Imbalance Factor
16 (192 × 192)	1.93	5.034
64 (96 × 96)	0.59	1.501
256 (48 × 48)	0.43	1.108
4096 (12 × 12)	0.4	1.017
36864 (4 × 4)	0.4	1.003
147456 (2 × 2)	0.47	1.017
589824 (1 × 1)	0.80	1.006

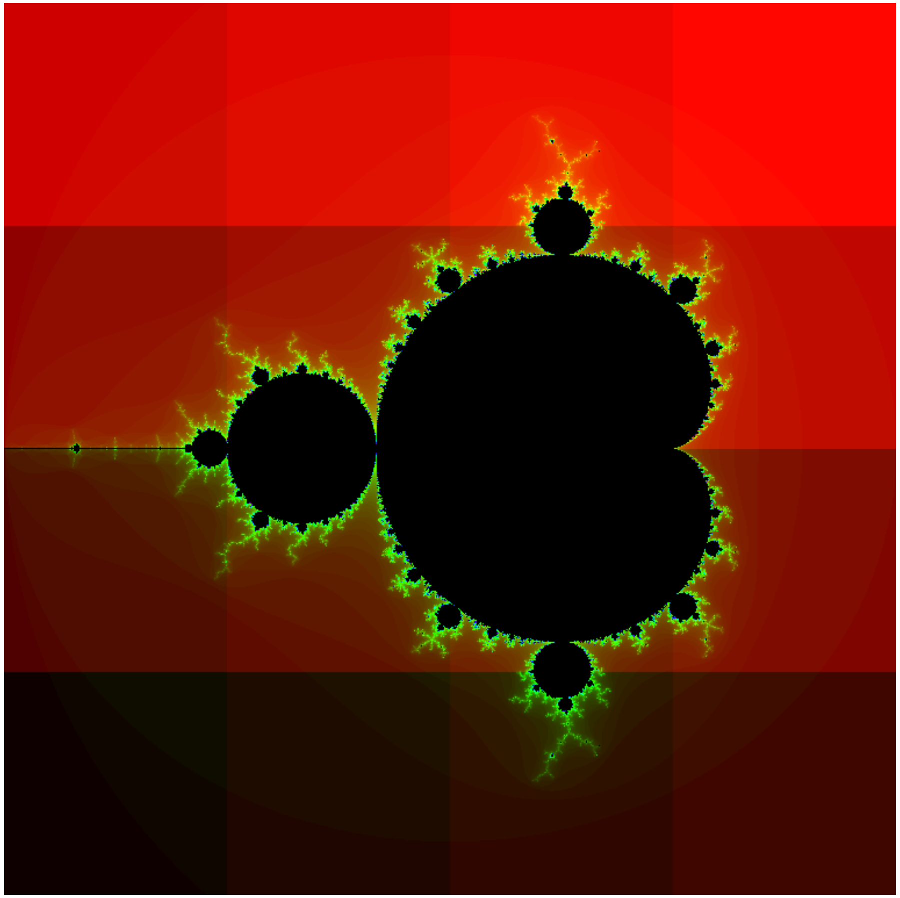
Table 2: Example execution Times for 16 workers and varying number of Tasks.

epcc

Results cont.



16 workers and 16 tasks



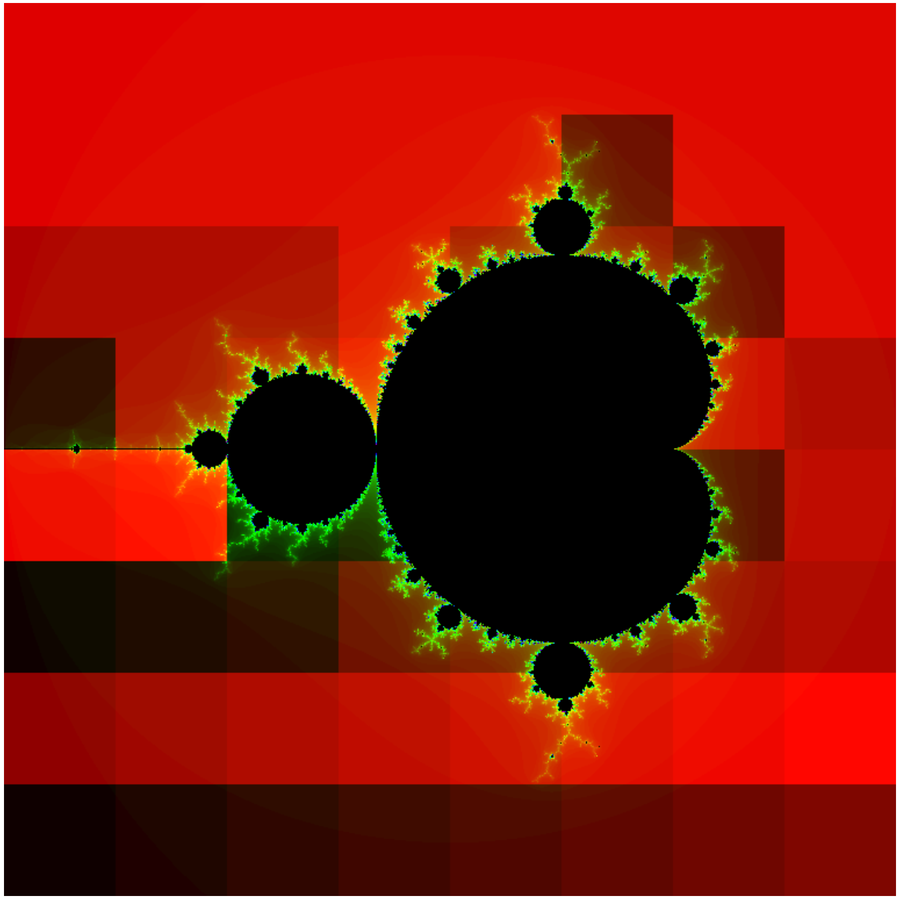
```
-----Workload Summary (number of
iterations)----

Total Number of Workers: 16
Total Number of Tasks:   16

Total   Worker Load: 498023053
Average Worker Load: 31126440
Maximum Worker Load: 156694685
Minimum Worker Load: 62822

Time taken by 16 workers was
1.929219 (secs)
Load Imbalance Factor: 5.034134
```

16 workers and 64 tasks



```
-----Workload Summary (number of
iterations)---------

Total Number of Workers: 16
Total Number of Tasks:   64

Total   Worker Load: 498023053
Average Worker Load: 31126440
Maximum Worker Load: 46743511
Minimum Worker Load: 10968369

Time taken by 16 workers was
0.586923 (secs)
Load Imbalance Factor: 1.501730
```

Key points to take away

TASK FARMS

- Also known as the master/worker pattern
- Allows a master process to distribute work to a set of worker processors.
- Can be used for other types of tasks but it complicates the situation and other patterns may be more suitable for implementing.
- Master process is responsible for creating, distributing and gathering the individual jobs.
- Can improve load balance by using more tasks than workers
  - with some overhead
- Load imbalance adversely affects performance
  - especially as number of processors increases

Key points to take away

TASKS

- Units of work
- Vary in size, do not have to be of consistent execution time. If execution times are known it can help with load balancing.

QUEUES

- Master generates a pool of tasks and puts them in a queue
- Workers assigned task from queue when idle

Key points to take away

LOAD BALANCING

- How a system determines how work or tasks are distributed across workers (processes or threads)
- Successful load balancing avoids idle processes and overloading single cores
- Poor load balancing leads to under-utilised cores, reducing performance.

Key points to take away

COST

- Increasingly important
- Finite budgets require optimal use of resources requested.
- Load balancing is just one method of ensuring optimal usage and avoiding wasting resources.
- More power and resources do not necessarily mean improved performance.
- Always ask – is it necessary to run this on 4000 cores or could it be run on 2000 more efficiently?